

Vortex Fluid Relaxation Model for Torsional Oscillation Responses of Solid ^4He

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A phenomenological model is developed to explain new sets of detailed torsional oscillator data for hcp ^4He . The model is based on Anderson's idea of a vortex fluid(vortex tangle) in solid ^4He . Utilizing a well-studied treatment of dynamics of quantized vortices we describe how the "local superfluid component" is involved in rotation(torsion oscillations) via a polarized vortices tangle. The polarization in the tangle appears both due to alignment of the remnant or thermal vortices and due to penetration of additional vortices into the volume. Both are supposed to occur in a relaxation manner and the inverse full relaxation time τ^{-1} is the sum of them. One of them is found to change linearly with respect to the rim velocity V_{ac} . The developed approach explains the behavior of both $NLRS$ and ΔQ^{-1} seen in the experiment. We reproduce not only the unique V_{ac} dependence, but also obtain new information about the vortices tangle, namely a divergence in τ at extrapolated $T \sim 30$ mK.

After the first report on "non-classical rotational inertia" (NCRI) in solid ^4He samples[1], the confirmation came from several torsional oscillation(TO) experiments, including some by the present authors [2]. This finding had been discussed in connection to the $NCRI$ [3] of a supersolid as originally proposed by Leggett[4]. The measured drop of the period is expected to appear due to the reduction of momentum of inertia, which originates in the appearance of a superfluid component which does not follow the rotation of the sample cell wall. It was, however, at problematically high T where the "transition" was reported. The following findings of the real onset temperature T_o [5] and possible vortex fluid(VF) state below T_o [5, 6], would overcome the too high T_c for BEC. And recent our work[7] demonstrated a transition from the VF state to the supersolid state(SS) in solid ^4He below about 75 mK. In the present work we discuss the vortex dynamics in the VF state, including the supercooled condition or measurements under "equilibrium conditions"[7]. On the other hand, there have been various measurements by now showing that this phenomenon depends on various parameters like pressure, impurity, sample quality[8], "orientation"[9] etc. Dependence of the phenomenon on the excitation amplitude of the TO (or on the rim velocity V_{ac}) gives rise to special interest as previously discussed[5]. Fig.1 shows our TO results for the V_{ac} dependence of both $NCRI$ (namely relative change of period $\Delta P/P$) and dissipation ΔQ^{-1} (inverse quality factor) of the 49 bar sample, the former(lower column) was given in our experimental work[5] and discussed as evidence for the VF state, pointing out the $\log V_{ac}$ linear dependence, originally proposed by Anderson[6]. In addition, the unique feature that signal decreases when V_{ac} is increased, is argued to be evidence for thermally excited vortices in the VF state. Let us discuss some properties of the observed phenomena[5] depicted in Fig.1. ①.It is easy to see that the V_{ac} dependence of the period drop disappears for some characteristic velocities

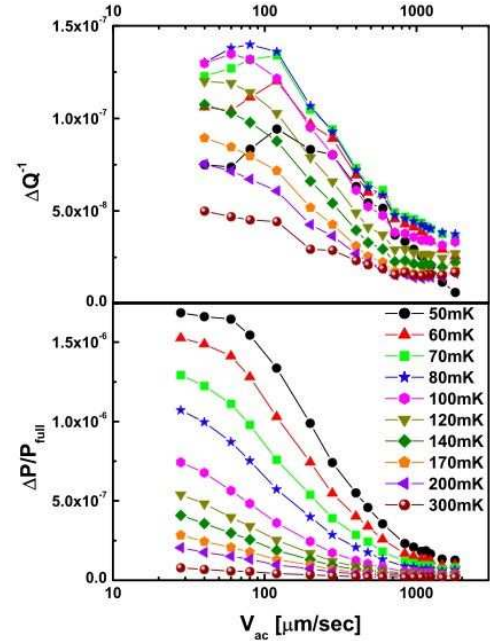


FIG. 1: New Data set of TO Responses throughout the vortex fluid state[5], including supercooled condition. Upper column indicate energy dissipation ΔQ^{-1} and the lower column shows nonlinear rotational susceptibility, $NLRS = \Delta P / \Delta P_{load}$ for 49 bar hcp ^4He at different T 's as functions of V_{ac} .

$V_{ac} \lesssim 10 - 30 \mu\text{m/s}$. ②.The drop of period (NLRS) decreases as the applied V_{ac} increases. This is a sign that the superfluid part is being gradually involved in rotation. ③.For steady rotation with velocities exceeding a characteristic velocity this effect vanishes, the sample rotates as a whole. ④.The characteristic value of the ratio $\frac{\Delta P}{P} / \Delta Q^{-1}$ is T and the pressure dependent quantity of order of unity. ⑤.At high T , the dissipation ΔQ^{-1} is a monotonically decreasing function of V_{ac} , whereas at low T there is an obvious maximum. ⑥.One more feature among reported results is the frequency dependence

of both P and ΔQ^{-1} [10]. In the literature there was a speculation that this behavior can be associated with quantized vortices. For instance, in [11] it is pointed out that velocity $V_{ac} \approx 10 \mu\text{m/s}$ coincides with the velocity created by a single circulation around the sample. Prokof'ev[4] pointed out that "To understand why NLRs decreases with V_{ac} , one has to consider the non-linear response of vortex loops and pinned vortex lines to the flow". Huse et al.[12] developed a simple phenomenological model which introduces dissipative relative motion of two components realized via "phase slips" of quantized vortices. P.W. Anderson[6] describes the scenario of a set of chaotic vortices (vortex fluid or vortex tangle) which under the torsional oscillation(TO) behaves like vortex-antivortex pairs in the Kosterlitz-Thouless model, and free (unbalanced) vortices bring the superfluid part into rotation. The role of vortices in rotation and torsional oscillation of solid helium had been discussed also in [13, 14].

In the following, we propose a phenomenological model describing the behavior of the torsional oscillations in the presence of a vortex tangle. In a vortex free sample or in the case of an absolutely isotropic vortex tangle, the superfluid fraction does not participate in rotation or torsional oscillations. Therefore the momentum of inertia I_{full} acquires a deficit $I_{SF} = \rho_s V R^2 / 2$ where ρ_s is the 'local' superfluid density for the VF state, and V is the volume of the sample. Angular momentum of the superfluid fraction appears only due to the presence of either aligned vortices (vortex array) or due to the polarized vortex tangle having nonzero total average polarization $\mathbf{P} = \mathcal{L} \langle \mathbf{s}'_z(\xi) \rangle$ along the applied angular velocity $\mathbf{\Omega}$ (axis z , the magnitude of $\Omega = V_{ac}/R$, where R is radius of the sample). Here \mathcal{L} is the vortex line density (total length per unit volume), $\mathbf{s}(\xi)$ is the vector line position as a function of label variable ξ , $\mathbf{s}'(\xi)$ is the tangent vector. In the *steady case* there is a strictly fixed relation between the total polarization $\mathcal{L} \langle \mathbf{s}'(\xi) \rangle$ and applied angular velocity $\mathbf{\Omega}$,

$$\mathbf{\Omega} = \kappa \mathbf{P} / 2 = \kappa \mathcal{L} \langle \mathbf{s}'(\xi) \rangle / 2. \quad (1)$$

Here κ is the quantum of circulation. In a case when vortex filaments form an array the quantity \mathcal{L} coincides with 2D density n and (1) transforms to the usual Feynman's rule. Angular momentum of the superfluid part can be written as $\mathbf{M}_{SF} = I_{SF} \mathbf{\Omega} = I_{SF} \kappa \mathbf{P} / 2$.

The situation drastically changes in a nonstationary (transient or oscillating) case. The total polarization $\mathbf{P}(t)$ changes in time owing to both the vortex line density $\mathcal{L}(t)$ and the mean local polarization $\langle \mathbf{s}'(t) \rangle$ change in time according to their own, relaxation-like dynamics. Therefore the angular momentum of the superfluid part is not $\mathbf{M}_{SF} = I_{SF} \mathbf{\Omega}$ anymore. Because of relaxation processes there is retardation between $\mathbf{\Omega}(t)$ and $\mathbf{M}_{SF}(t)$, and the connection between them is nonlocal in time and $\mathbf{M}_{SF}(t)$ is some functional of time dependent angular

velocity $\mathbf{\Omega}(t)$. There are two possible mechanisms for relaxation-like polarization of the vortex fluid. The first is an alignment of elements of the vortex lines due to interaction with the normal component (See [15] for detailed explanations). This interaction (mutual friction) is proportional to the local normal velocity, which in turn is proportional to the rim velocity \mathbf{V}_{ac} . Thus it is natural to suppose that polarization \mathbf{P} of the vortex tangle due to alignment of filaments along $\mathbf{\Omega}(t)$ occurs with typical inverse time $\tau_1^{-1}(\mathbf{V}_{ac})$ which is proportional to the rim velocity \mathbf{V}_{ac} . Let us illustrate the above with consideration performed in [15]. In the presence of mutual friction there is a torque acting on the line and the angle ϕ between axis z and the line element changes according to the equation $d\phi/dt = \alpha(\mathbf{V}_{ac}/R) \sin \phi$ (α is the friction coefficient, dependent, in general, on T and pressure p). Except for a short transient, the solution to this equation can be described as a pure exponential $\sim \exp(-t/\tau_1(\mathbf{V}_{ac}))$, with the velocity dependent inverse time $\tau_1^{-1}(\mathbf{V}_{ac}) \sim \alpha \mathbf{V}_{ac}/R$. Thus, we conclude that during time-varying rotation or torsional oscillation vortex filaments tend to align along the angular velocity direction. However, there can be not enough pre-existing vortex lines in the tangle to involve all the superfluid part into the rotation to satisfy the relation (1), or on the contrary the initial vortex tangle can be excessively dense. In this case deficient (extra) vortices should penetrate into (leave from) the bulk of the sample. This penetration occurs in a diffusion-like manner[16] and leads to the relaxation-like saturation of the vortex line density $\mathcal{L}(t)$. We assume that this saturation occurs in an exponential manner with some characteristic inverse time $\tau_2^{-1} = \beta$. Due to linearity of the diffusion process we suppose that coefficient β is velocity independent, but can be a function of T and p . Combining both mechanisms we assume that the whole polarization of the vortex fluid occurs in the relaxation manner with pure exponential behavior $\varphi(t'/\tau) \sim \exp(t'/\tau)$, and the inverse time τ^{-1} of relaxation is just the sum of $\tau_1^{-1}(\mathbf{V}_{ac})$ and τ_2^{-1} ,

$$\tau^{-1} = \alpha(T) \mathbf{V}_{ac}/R + \beta(T) \quad (2)$$

In the presence of relaxation the angular momentum $\mathbf{M}(t)$ of the superfluid part is related to the applied angular velocity $\mathbf{\Omega}(t)$ by the nonlocal relation,

$$\mathbf{M} = a \mathbf{\Omega}(t) + b \int_0^\infty \mathbf{\Omega}(t-t') \varphi\left(\frac{t'}{\tau}\right) \frac{dt'}{\tau}. \quad (3)$$

Relation (3) implies that the angular momentum $\mathbf{M}(t)$ depends on the applied angular velocity $\mathbf{\Omega}(t)$ taken in the all previous moments of time with the weight $\exp(-t'/\tau)$. To clarify the physical meaning of constants a and b we consider the limiting cases of very small and very large frequencies. In case $\omega \rightarrow 0$ the slowly changing function $\mathbf{\Omega}(t-t')$ can be considered as a constant and be

taken out of the integral, whereupon the rest of integral becomes unity and we have $\mathbf{M}_{\omega \rightarrow 0} = (a + b)\mathbf{\Omega}$. But at the same time, both components participate in the solid body rotation, thus $(a + b) = I_{full}$. In the opposite case of very large frequencies, $\omega \rightarrow \infty$, the integral from rapidly oscillating functions $\mathbf{\Omega}(t - t')$ vanishes, so $\mathbf{M}_{\omega \rightarrow \infty} = a\mathbf{\Omega}$. Since under these conditions the superfluid component does not participate in the motion at all, we conclude that the constant a is nothing but the full moment of inertia I_N of the sample without the superfluid part (which includes momentum of inertia of the empty cell I_{empty}). Thus, the quantity b is moment of inertia I_{SF} of the superfluid part. Substituting (3) with $a = I_N$ and $b = I_{SF}$ into the equation of motion of the TO, we get

$$\frac{d}{dt} \left[I_N \mathbf{\Omega}(t) + I_{SF} \int_0^\infty \mathbf{\Omega}(t - t') \varphi\left(\frac{t'}{\tau}\right) \frac{dt'}{\tau} \right] + k\theta = 0. \quad (4)$$

Here $\theta(t)$ is the angle of rotation of the oscillator, k is the spring constant. Relation(4) is an integro-differential equation and, in general, not easy to solve. Because $\varphi(\frac{t'}{\tau})$ is a pure exponential function we can eliminate the integral term. Omitting details we arrive at the case where equation (4) is reduced to an ordinary differential equation of the third order, which has a solution in the form $\theta(t) = \theta_0 \exp(i\omega t)$. The frequency ω satisfies the relation

$$\omega = \sqrt{\frac{k}{I_{full}}} \left(1 + \frac{I_{SF}}{2I_{full}} \frac{(\omega\tau)^2}{(\omega\tau)^2 + 1} + \frac{I_{SF}}{2I_{full}} \frac{i\omega\tau}{(\omega\tau)^2 + 1} \right).$$

Thus, the frequency of the oscillation consists of three parts. The first one $\omega_0 = \sqrt{k/I_{full}}$ describes the oscillation with full moment of the inertia I_{full} as if all ingredients (empty cell, normal part, superfluid part) fully participate in motion. The second term is responsible for increase of the frequency because the superfluid component participates in the torsional oscillation only partly. The third term is the imaginary one. It describes the attenuation of the oscillation amplitude, i.e. it describes the dissipation. The amplitude decreases (with time) as $\exp[-\Im(\omega)t]$, and the inverse quality factor is $Q^{-1} = \frac{2\Im(\omega)}{\omega}$. Using the smallness of the $I_{SF} \ll I_{full}$ we put $\omega = \omega_{full}$ in the right hand side, yielding (index in ω_{full} is omitted)

$$\frac{\Delta P}{P} = -\frac{1}{2} \frac{I_{SF}}{I_{full}} \frac{(\omega\tau)^2}{(\omega\tau)^2 + 1}. \quad (5)$$

$$\Delta Q^{-1} = \frac{2\Im(\omega)}{\omega} = \frac{I_{SF}}{I_{full}} \frac{(\tau\omega)}{\tau^2\omega^2 + 1} \quad (6)$$

Relations (5),(6) are the final solution to the problem of the torsional oscillation when the superfluid component is involved in rotation via polarized vortex fluids, and polarization occurs in the relaxation-like manner.

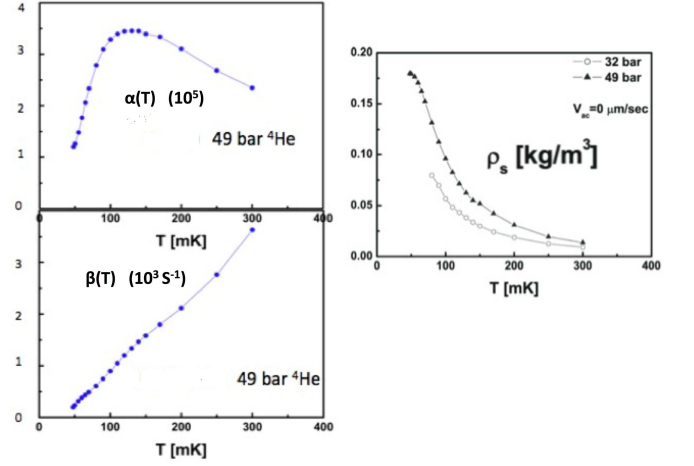


FIG. 2: Parameters $\alpha(T)$, $\beta(T)$, and $\rho_s(T)$ obtained from the data of Fig.1 with the use of analysis described in text. $\beta(T)$ goes to zero, or τ to infinity at extrapolated $T \approx 30$ mK.

Being phenomenological, the approach developed does not allow determining some quantities entering the formalism. Thus the parameters $\alpha(T, p)$ and $\beta(T, p)$ responsible for the relaxation of the vortex tangle should be also obtained on the basis of the approach describing dynamics of quantized vortices, which is so far absent. Nevertheless comparison of our results with the experimental data allows us to explain a series of experimental results and to get some quantitative information and insights. Let us analyze relations (5) and (6). From relations (5),(6) of our paper it follows that $\frac{\Delta P}{P} / \Delta Q^{-1}$ is equal to $(1/2)(\omega\tau)$. It can take any value depending on the arrangement of the experiment. But usually observations should be under conditions with $\omega\tau$ on the order of unity. Therefore in many experiments the $\Delta P/P$ and ΔQ^{-1} are of the same order of magnitude, although sometimes they can be significantly different (see [12]). Dividing the first relation by the second one and taking the zero \mathbf{V}_{ac} limit in the relation $\frac{\Delta P}{P} / \Delta Q^{-1} = (1/2)(\omega\tau)$ we get an expression for relaxation time $\beta(T)$ due to diffusion of vortices. Taking further the zero \mathbf{V}_{ac} limit for the period drop, and assuming that $\beta(T)$ abruptly vanishes below the 'critical velocity' (which is equivalent to absence of vortices), we find the superfluid momentum of inertia I_{SF} , and, consequently superfluid density ρ_s can be extracted from the graphs for $\Delta P/P$. Knowing $I_{SF}(\rho_s(T))$, $\beta(T)$ and fitting the curves $\Delta P/P$ as functions of \mathbf{V}_{ac} it is possible to determine the inverse relaxation time due to aligning $\tau_1^{-1}(\mathbf{V}_{ac}) \sim \alpha(T)\mathbf{V}_{ac}/R$ and quantity $\alpha(T)$. Performing all procedures described above, we have all the necessary data, as shown in Fig. 2 where parameters $\alpha(T)$, $\beta(T)$, and $\rho_s(T)$ are depicted.

In Fig. 3 we show ΔQ^{-1} and $\frac{\Delta P}{P} = NLRS$ as functions of \mathbf{V}_{ac} , drawn using relations (5) and (6) and extracted experimental data. It can be seen that shapes of curves

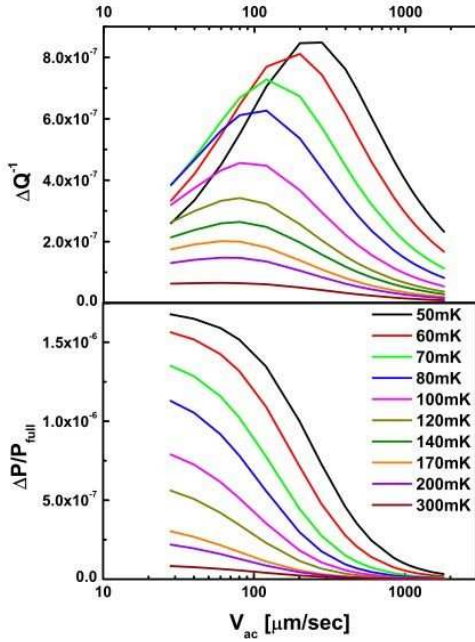


FIG. 3: Energy dissipation ΔQ^{-1} and nonlinear rotational susceptibility $NLRS$ at different T as a function of V_{ac} , obtained using relations (5),(6) with parameters taken from Fig. 2

and their response to the change of T correspond to the curves shown in Fig.1 and Fig.2. It is seen that in the limit $V_{ac} \rightarrow 0$, or $\omega \rightarrow \infty$, or $\alpha(T) \rightarrow 0$, $NLRS$ reaches the maximum value. Physically it is clear, since under these conditions the superfluid part cannot participate in rotation at all. Other limits $V_{ac} \rightarrow \infty$, or $\omega \rightarrow 0$, or $\alpha(T) \rightarrow \infty$ correspond to the vanishing of the effect, which is also reasonable since under these conditions the superfluid part participates in the solid body rotation, and no effect appears. If relaxation due to diffusion (penetration) is weak, then the dependence of dissipation becomes non-monotonic. Analysis shows that the critical value of $\tau_2^{-1} = \beta(T)$ is equal to the frequency ω . In fact, this can differ by some factor on the order of unity. One of the possible reasons for this difference is that we calculate using a purely exponential relaxation process, whereas in reality it can be better described by a more complicated dependence. The maximum value of dissipation ΔQ_{peak}^{-1} should be at $\frac{1}{2} \frac{\Delta P}{P}$ and it should be reached at values of the rim velocity $V_{ac} = R(\omega - \beta(T))/\alpha(T)$. This tendency is easily seen in Fig. 1, ΔQ_{peak}^{-1} decreases with T and shifts in the direction of small V_{ac} , then for some "critical temperature" when $\tau_2^{-1} = \beta(T)$, the peak disappears entirely. It happens at T about 120 mK. Comparing with the experimental data one can conclude that the behavior described above indeed takes place for T above about 75 mK, but the agreement fails for lower T . It is remarkable that 75 mK was detected by authors of the present paper, as T_c below which a hysteretic behavior takes place as a sign of a transition to a supersolid(SS) state(see [7]). Relations (5) and (6) can also explain the

$f = \omega/2\pi$ dependence of $NLRS$ and ΔQ^{-1} observed in [10]. Indeed, the significant dependence on ω appears when the inverse time τ^{-1} of relaxation is comparable with ω , which can happen at higher T . In this range of parameters, the $\Delta P/P$ (5) is a monotonic function of ω . In summary the phenomenological model of relaxation processes of the VF state has been introduced. Unsteady rotation and torsional oscillation have been studied. Dependence of both the $NLRS$ and the ΔQ^{-1} on T , V_{ac} and f have been studied. The results obtained may serve as a good qualitative description of the corresponding measurements in the VF state in solid ^4He . Combining theoretical predictions with experimental data it became possible to obtain some quantitative results. Actually recent experimental results[17] can be well understood in terms of the present VF analysis, as an alternative to the interpretation in terms of superglass, by other authors.

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- [1] E. Kim and M. H.W. Chan, Nature (London) 427, 225 (2004); Science 305, 1941 (2004); Phys. Rev. Lett. 97, 115302 (2006)
 - [2] M. Kondo, S. Takada, Y. Shibayama, and K. Shirahama, J. Low Temp. Phys. 148, 695 (2007); A. S. C. Rittner and J. D. Reppy, Phys. Rev. Lett. 97, 165301 (2006); A. Penzev, Y. Yasuta, and M. Kubota, J. Low Temp. Phys. 148, 677 (2007).
 - [3] A. J. Leggett, Phys. Rev. Lett. 25, 1543 (1970).
 - [4] see reviews, for example, D. Galli and L. Reatto, J. Phys. Soc. Jpn. 77, (2008) 111010; S. Balibar and F. Caupin, J. Phys.:Condens. Matter 20, 173201 (2008); N. Prokofev, Advances in Physics 56, 381 (2007).
 - [5] A. Penzev, Y. Yasuta, and M. Kubota, Phys. Rev. Lett. 101, 065301 (2008).
 - [6] P.W. Anderson, Nature Phys. 3, 160 (2007); arXiv:0705.1174; Phys. Rev. Lett. 100, 215301 (2008).
 - [7] N. Shimizu, Y. Yasuta, and M. Kubota, arXiv:0903.1326, submitted to Phys. Rev. Lett.
 - [8] A.S.C. Rittner and J.D. Reppy, Phys. Rev. Lett. 101, 155301 (2008).
 - [9] J.T. West, *et al.*, Phys. Rev. Lett. 102, 185302 (2009).
 - [10] Y. Aoki, J. Graves, and H. Kojima, Phys. Rev. Lett. 99, 015301 (2007).
 - [11] E. Kim and M. H.W. Chan, Science 305, 1941 (2004).
 - [12] David A. Huse and Zuhair U. Khandker, Phys. Rev. B 75, 212504 (2007).
 - [13] Wayne M. Saslow, Phys. Rev. B 71, 092502 (2005).
 - [14] Y. Pomeau and S. Rica, Phys. Rev. Lett. 72, 2426 (1994).
 - [15] M. Tsubota *et al.*, Phys. Rev. B 69, 134515 (2004).
 - [16] Sergey K. Nemirovskii, arXiv:0902.3720v1.
 - [17] B. Hunt, *et al.*, Science 324, 632 (2009).